HITS: Hurricane Intensity and Track Simulator with North Atlantic Ocean Applications for Risk Assessment

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(Manuscript received 9 June 2014, in final form 16 March 2015)

ABSTRACT

A nonparametric stochastic model is developed and tested for the simulation of tropical cyclone tracks. Tropical cyclone tracks demonstrate continuity and memory over many time and space steps. Clusters of tracks can be coherent, and the separation between clusters may be marked by geographical locations where groups of tracks diverge as a result of the physics of the underlying process. Consequently, their evolution may be non-Markovian. Markovian simulation models, as are often used, may produce tracks that potentially diverge or lose memory quicker than happens in nature. This is addressed here through a model that simulates tracks by randomly sampling track segments of varying length, selected from historical tracks. For performance evaluation, a spatial grid is imposed on the domain of interest. For each grid box, long-term tropical cyclone risk is assessed through the annual probability distributions of the number of storm hours, landfalls, winds, and other statistics. Total storm length is determined at birth by local distribution, and movement to other tropical cyclone segments by distance to neighbor tracks, comparative vector, and age of track. The model is also applied to the conditional simulation of hurricane tracks from specific positions for hurricanes that were not included in the model fitting so as to see whether the probabilistic coverage intervals properly cover the subsequent track. Consequently, tests of both the long-term probability distributions of hurricane landfall and of event simulations from the model are provided.

1. Introduction

Extreme weather events such as tropical cyclones occur with low frequency. Because of the low probability of North Atlantic Ocean tropical cyclone landfall, landfall statistics are difficult to estimate. Statistical methods can be employed to resample the historical data, creating a large number of tracks used to improve estimates of the probability extremes. These statistics are useful for insurance companies in determining premiums, and for communities and governments in determining disaster plans and building codes.

Several models have been introduced to estimate tropical cyclone landfall probabilities. Some directly simulate tropical cyclone landfall and intensity by Monte Carlo methods (Clark 1986; Chu and Wang 1998), dimensionality reduction (Buchman et al. 2011), or by fitting a probability function (Emanuel and Jagger 2010), while others create sets of simulated tracks (Casson and Coles 2000; Emanuel et al. 2006; Hall and Jewson 2007; T. M. Hall and S. Jewson 2008, unpublished manuscript, available at http://arxiv.org/pdf/0801.1013v1.pdf; Hallegate 2007; Rumpf et al. 2007, 2009; Vickery et al. 2000; Yonekura and Hall 2011).

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DOI: 10.1175/JAMC-D-14-0141.1

Vickery et al. (2000) used a 5° × 5° box to determine track heading, speed, and intensity, through a regression of historical storm data on location attributes, prior time step storm speed, and direction. Hurricane intensity is modeled as a function of prior intensity for up to three time steps and of sea surface temperature at appropriate locations. A random innovation consistent with the regression model is added to generate the tracks. A number of other parameters are also estimated within a regression framework, and landfall probability distributions are estimated from the simulations. The variance explained by their regression models is typically not high, but the conditional probability distributions derived generally look plausible. Casson and Coles (2000) simulate tracks by the shift of a uniform random value less than 100 n mi (1 n mi = 1.852 km) and use a simulated pressure plus land effects to produce a simulated wind speed. Emanuel et al. (2006) present a smoothed Markov chain model and beta and advection model using synthetic flow at 850 and 250 hPa. Hallegate (2007) used the later to investigate landfall and damage probabilities. In our work in the late 1990s, a Markov chain model was initially implemented to simulate these popular paths with transition probabilities changing from grid box to grid box, but it was found to be too diffusive. For each grid box, the one-step transition probability to adjoining grid boxes and to the grid box (5°) itself was estimated. The Markov chain model was fit to the post–airplane reconnaissance historical record (1944–99) and then track simulations were generated, seeded by a random selection of historical track birth positions. A comparison of the historical storm residence time in each grid box with that simulated is presented in Fig. 1. The match is rather poor, reflecting the diffusion one qualitatively expects from a "lag one" space–time model, relative to a model that considers "runs" or longer-term persistence. Smoothing the Markov chain probabilities, as was done by Emanuel et al. (2006), improves the estimates of the probabilities of transition, by reducing the variance, but does not change the diffusive behavior. The Markov chain model is also sensitive to the grid scale chosen for parameterization.

Recognition of these issues with a spatial Markov chain model led to alternate models. Hall and Jewson (2007), T. M. Hall and S. Jewson (2008, unpublished manuscript, available at http://arxiv.org/pdf/0801.1013v1.pdf), and Yonekura and Hall (2011) use a bootstrap simulation of the birth positions to select a birth location, a regression on data from neighboring tracks is used to choose a direction of movement for the next 6 h, and a track termination criterion is imposed. Consequently, this is a Markovian model in time, but with spatial movements that are determined by a set of covariates via local regression, and a random component that is based on a

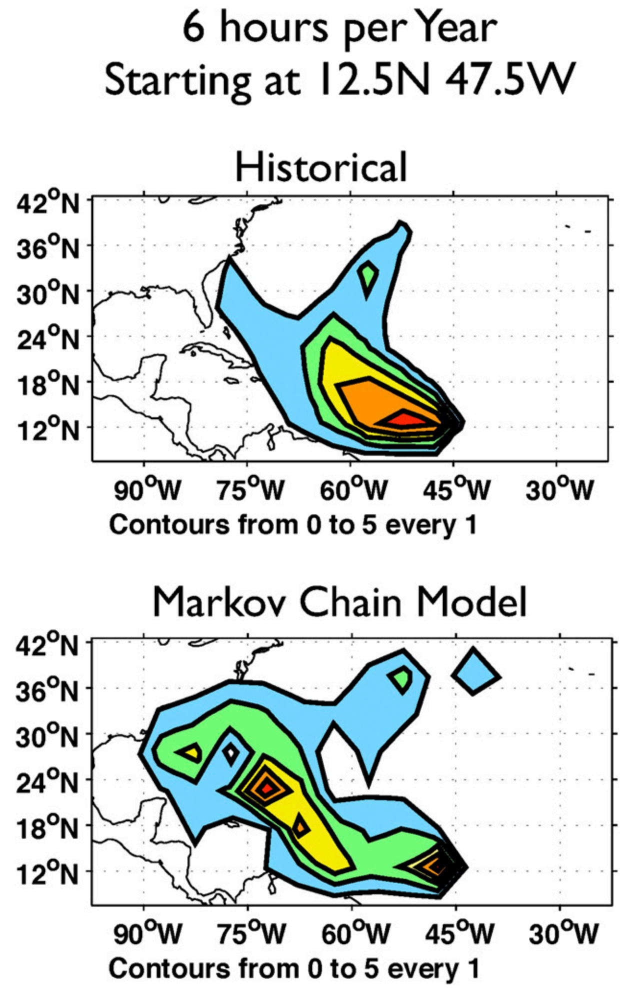

FIG. 1. The 1944–99 6-h time periods per year for cyclones starting in the box covering 10°–15°N and 45°–50°W for the (top) historical record and (bottom) the Markov chain model.

multivariate Gaussian distribution. The Markov-in-time process using the native data resolution allows the building of tracks every 6 h, with local decisions as to movement. The alternate models demonstrate improvements over the spatial Markov chain model.

Rumpf et al. (2007, 2009) first group the tracks into four and six clusters for the Pacific Ocean (2007) and Atlantic (2009), respectively. For each cluster, they simulate cyclone starts by employing a Poisson point process using a nearest-neighbor intensity estimate. They then consider a random walk model applied to the 6-hourly data from 1945 to 2004. Next, they randomly sample a direction of movement, a translation speed, and a wind speed through local fitting using a kernel estimator, as well as making a movement for each 6-h step. They continue the random walk and at each step they make a decision based on a stopping probability that is also estimated locally. They check whether the generated storm conforms to the cluster that it was generated from and use this to accept or reject the storm that was generated. Conceptually, their approach is then similar to that of Hall and Jewson but with a different implementation where kernel density estimators and classification do much of the work. In summary, the simulation models presented in the literature are by and large Markovian, looking at ways to march the track one step (in time and/or gridded space) at a time based on conditional probabilities inferred from track and climate attributes. The more successful models appear to consider transition probabilities or conditional probability distributions that change with spatial location and with respect to exogenous attributes, but not with respect to the age of the track, or the extended past history of the track.

To a first-order approximation, tropical cyclones move in the direction that the winds (over the depth of the storm) steer them. In the northern Atlantic, the northeasterly trade winds move the storms westward from the African coast. The prevailing flow around the subtropical high curves them, and other cyclones generated in the Caribbean Sea and Gulf of Mexico, northward approaching the North American coast and then eastward in the middle latitudes. Elsner and Kara (1999) call this a parabolic sweep. The position and strength of the subtropical high, the extratropical circulation, and the birth location of the tropical cyclone varies, allowing variations in the tracks. To reflect the parabolic sweep, and variations of it, a spatial model is needed. More ''popular'' track paths have a higher probability of occurring than do ''unpopular'' ones.

Considering higher dependence (e.g., to the two prior steps) in a Markov model leads to an explosion in the number of parameters to be estimated for the resulting transition probability matrix, which is commonly called the ''curse of dimensionality'' and is hence not indicated with the hurricane track dataset. In the time series modeling literature, the situation is often addressed by considering a semi-Markov or Markov renewal model (Bhat and Miller 1972; Çinlar 1969, 1975; Gilbert et al. 1972; Foufoula-Georgiou and Lettenmaier 1987). A direct application of the Markov renewal model to the tropical cyclone track setting is not obvious at first glance, since one needs a specification of discrete states for the system, prior to modeling the conditional distribution of the time to be spent in each future state. Inspired by the Markov renewal idea, we propose a modeling strategy where we consider the time t_i to be spent along a candidate track i to be a random variable and we allow the selection of the candidate track and the associated t_i to depend on location and other attributes. This provides the basis for the hurricane intensity and track simulator (HITS) model presented in this paper.

The historical hurricane record is discussed in section 2, along with 2012 data used for model verification on novel tracks. Section 3 describes the HITS algorithm and section 4 presents the results of the comparison to the historical record using percentiles, and a visual comparison of distributions with split violin plots. The last section discusses results, a brief comparison to other hurricane track models, and plans for future work.

2. Data

The data used for the model are taken from the historical best-track North Atlantic hurricane dataset (HURDAT; Jarvinen et al. 1984) from 1851 to 2011 with information on storm position (latitude and longitude) and wind speed every 6 h. The data were obtained from the National Hurricane Center, and the record contains ''named'' storms, defined as storms whose maximum sustained (1-min-averaging period) surface winds exceeded 18 m s^{-1} (34 kt, or 39 mi h^{-1}). The primary table of tropical cyclone data consists of the number of storms in a year, the catalog storm number, the year, month, day, and position on the track (6-h time step), the latitude, longitude, and wind speed (kt). A domain of 5°–45°N and 100°–25°W was chosen and segmented by a 5° × 5° grid to give 120 boxes. These grids are used to report model performance evaluation, but are not used in model fitting. The model is built directly using the track data without spatial discretization. During the 161-yr record, 1465 tropical cyclones spent all or part of their lifetimes in the domain.

As a result of routine aircraft reconnaissance missions into tropical cyclones beginning in 1944, details on the position of the tropical cyclone eye are available. This has led to greater accuracy in the 6-h position data in storms far from land or shipping lanes. The storm durations in the prior period (1851–1943) are shorter than in the subsequent period (1944–2011) as a result of this change in observing method. The birth location of the storms is sampled from the post-1944 data. However, all available track data are used for neighbors to model tropical cyclone movement behavior, following a philosophy of nonideal data inclusion (Halevy et al. 2009). HITS based on the historical data was used to simulate tracks of recent storms (not included in the model-fitting set) to test how well the conditional simulations from a particular position of a hurricane provide coverage of the actual hurricane track from that point on. This is a stronger test of the algorithm than reported by any of the previous models.

3. HITS conceptual model

Çinlar (1969) provides a formal introduction to the Markov renewal model. Consider a finite number of states, for example, wet or dry for rainfall. In a Markov chain model, state transitions occur at a fixed time step (e.g., daily), and the parameter of interest is the state transition probability matrix for that time step. In a Markov renewal process, the time spent (e.g., wet or dry spell length) in each state can depend on the time spent in the prior state. Hence, the key parameter is the conditional probability distribution of the time to spend in the new state, given the time spent in the previous state (e.g., the wet spell duration depends on the previous dry spell duration), or $f(\tau_k \mid \tau_l)$, where l is the current state, k is the new state, and τ_l and τ_k are the corresponding durations. A generalization of the Markov renewal model to a nonhomogeneous Markov renewal model can be obtained if the conditional probability distributions of the durations in each state are further allowed to depend on covariates at the time of state transition. For instance, the covariates could be the calendar month, or an atmospheric circulation index (for the rainfall example) such as the Pacific–North America index. In this case the conditional probability distribution for the model is $f[\tau_k \mid \tau_l, \boldsymbol{\theta}(t)]$, where $\boldsymbol{\theta}(t)$ represents a vector of covariates at the time t, when the transition (l, k) takes place.

Hurricane tracks are often well organized in a region. For instance, Nakamura et al. (2009) used the k-means algorithm with the geometry of hurricane tracks to identify six clusters associated with Atlantic hurricanes. One could consider each of these clusters as a state, and consider the development of a renewal model for the transition of a hurricane across these clusters. However, this not led to a conceptually or practically attractive model for simulating hurricane tracks. At the same time, we recognize that while a hard clustering model, such as k-means, would assign each hurricane track to a specific cluster, a hierarchical clustering model may reveal a different, nested organizational structure for the tracks with clusters and subclusters, and a probabilistic clustering model would only assign a probability for each track to belong to a specific cluster. Further, if we consider state transitions across clusters, we are essentially considering tracks that originated in one cluster to migrate to another cluster, and so forth. Indeed, in the Atlantic we see this phenomenon. Tracks that originate in the eastern equatorial Atlantic can curve northward, continue to landfall, or curve southward, as they approach the continental landmass. From a hierarchical clustering perspective, these would represent the subclusters of perhaps a lower-level cluster, and the possibility of transition across these subclusters exists, especially at certain geographical regions, and/or at certain times into the trajectory.

Given these observations, we consider the following approach. Instead of explicitly identifying clusters of hurricane tracks at the outset, we consider the possibility that the observed hurricane tracks are stochastic realizations of possible tracks that could occur under a particular state of a hurricane-generating process. These states are latent or unobserved by us, but intuitively they correspond to the clusters or subclusters we try to identify from observed hurricane data. An observed hurricane track would then be a realization of a sequence of transitions between these latent states. In other words, a hurricane track could be born in a state associated with genesis in the eastern equatorial Atlantic, evolve as per this state's dynamics for a certain number of time steps, and then undergo a state transition to a latent state that conforms to curving north, curving south, or proceeding to landfall. This process could then repeat until a complete track is realized. A similar concept underlies the hidden Markov model (Hughes et al. 1999; Robertson et al. 2004) that is often used for downscaling precipitation from climate models. Latent states that govern regional precipitation dynamics and their transitions on a daily time step are identified based on the precipitation time series from multiple stations. The state transition probability could depend on the geographical location, as well as other attributes such as the wind shear, the surface temperature field, the state of ENSO or NAO, or other covariates. This corresponds to the nonhomogeneous hidden Markov model (Mehrotra and Sharma 2005; Kwon et al. 2009) for precipitation and, in our case, a nonhomogeneous hidden Markov renewal model (NHMRM).

The practical implementation of this idea into a space–time simulator for hurricane tracks is described next. A nonparametric approach based on k-nearest neighbor density estimation is used to develop the conditional simulation strategy implied by the NHMRM. Note that we do not try to formally estimate a parametric model for the NHMRM, but devise a resampling strategy that allows the construction of new simulated tracks assuming that each track segment is a realization from a latent state of the hurricane-generating process, and that the identification of the next track segment to resample corresponds to sampling from an underlying state transition.

To introduce some notation, laid out in the appendix, let us consider a latent state i, a current position $\mathbf{x}^*$, and a historical hurricane track $C(\mathbf{x}^*)$ that passes through $\mathbf{x}^*$. Now consider $C_{B(\mathbf{x}^*)}$ as the set of all historical hurricane tracks that pass through a region $B(\mathbf{x}^*)$ of a certain radius centered around $\mathbf{x}^*$. Each of the tracks in $C_{B(\mathbf{x}^*)}$ corresponds to a different latent state associated with the hurricane process, and based on its proximity is considered a candidate realization of a transition to that latent state. Hence, if during the simulation, we consider a shift from a track $C(\mathbf{x}^*)$ to one of the tracks in the set $C_{B(\mathbf{x}^*)}$ that would correspond to a transition from a latent state l to any of the other latent states, including l, since other tracks can also conform to the same underlying state. We consider that the probability of such a transition depends on specific geometrical and other attributes of each track in the set $C_{B(\mathbf{x}^*)}$.

Since, we are interested in simulating hurricane tracks under NHMRM, but not necessarily in identifying the latent states as part of the process, we can consider a simulated track $C(\mathbf{x})$ as a random curve, whose pieces are determined by successive transitions across historical tracks at a sequence of randomly selected candidate locations, an example of which is $\mathbf{x}^*$. Once a transition occurs, as per the renewal model, the time to spend, $\tau[C(\mathbf{x}^*)]$, in a realization from that latent state, that is, along the newly chosen track, $C(\mathbf{x}^*)$, needs to be simulated. This defines a new position $\mathbf{x}^*$ that is τ steps beyond the previous location, and the process is then repeated. The evolution of $C(\mathbf{x})$ in space and time may then be represented through a conditional probability distribution:

$$f\{C(\mathbf{x}^*), \tau[C(\mathbf{x}^*)] \,|\, \boldsymbol{\theta}(\mathbf{x}^*)\},$$

where $\boldsymbol{\theta}(\mathbf{x}^*)$ represents a set of covariates at the location $\mathbf{x}^*$ that includes attributes of the current state as represented by the hurricane track arriving at $\mathbf{x}^*$. Thus, given a certain location, the model considers the selection of a state going forward from that location, and the time that is to be spent following that track. This permits one to consider persistence of motion along tracks and avoids the diffusion associated with the one-step Markov chain models.

The $\boldsymbol{\theta}(\mathbf{x}^*)$ is a set of parameters relevant to the tropical cyclone process that vary by location. The $\boldsymbol{\theta}(\mathbf{x}^*)$ may include, for instance, the geographical location of $\mathbf{x}^*$, the index and direction vector of the track $C(\mathbf{x}^*) = i$ that was traveled to reach the location $\mathbf{x}^*$, atmospheric variables that influence track selection, and large-scale climate variables, such as sea surface temperatures or indices such as the Niño-3.4 or NAO. The specific choices for $\boldsymbol{\theta}(\mathbf{x}^*)$ made in the application presented here, and the nonparametric estimation of the conditional probability density function $f\{C(\mathbf{x}^*), \tau[C(\mathbf{x}^*)] \,|\, \boldsymbol{\theta}(\mathbf{x}^*)\}$, which is used to simulate the tracks, are discussed below as part of the algorithm presentation.

The HITS algorithm for simulating tropical cyclone tracks steps through time in 6-h intervals, making decisions along the path as to which historical track to follow. This process repeats itself until the lifetime is met. The total duration of a hurricane is also a model parameter that is randomly sampled in an initial step. A schematic of the process is provided in Fig. 2.

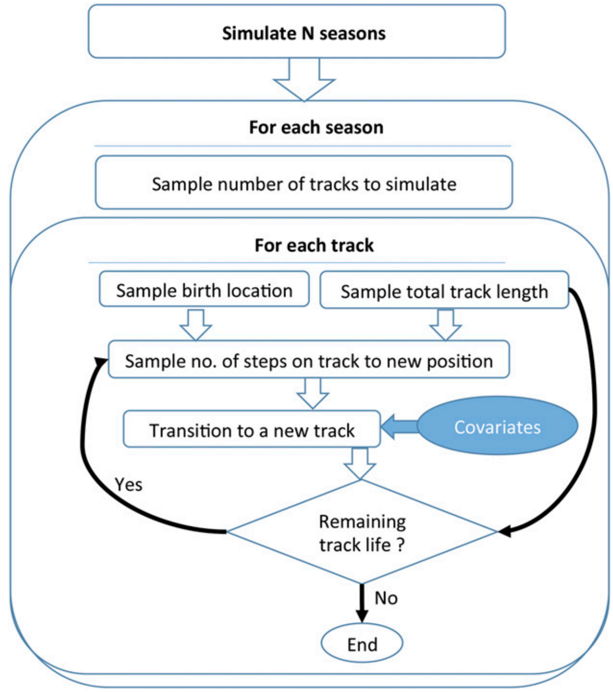

FIG. 2. Abbreviated flowchart of the HITS algorithm presenting an illustration of the steps in the process.

HITS algorithm

The implementation of the nonparametric resampling-based algorithm for the application of the NHMRM to the Atlantic sector is presented below. The state variables of the model define the latent states: the time spent in each state. The latent states are manifest in a realization as segments of historical hurricane tracks. In addition, the genesis location of each track simulated, the total life of each track, and the number of tracks to simulate for each season are all random variables.

The associated data and the simulation code are available from the authors. We consider the simulation of a hurricane season at a time, and can generate as many hurricane seasons as desired. For each simulated hurricane season, the following steps are taken:

1) Select the number of storms for the season:

$$N \sim U(N_l, N_u), \tag{1}$$

where N is the number of storms in a simulated season, N_l is the lowest number, and N_u is the highest number of storms during the years 1944–2011, while $U(\,)$ is a uniform draw, a bootstrap of the historical counts per year (Efron 1979). This can be conditioned on the observed or modeled large-scale climate state to reflect the dependence of number of tropical cyclone births on ENSO or other climate states. However, in the work presented here, we do not consider such a dependence.

2) For each potential storm, randomly select a birth location (first track position) p_b from all historical births post-1944 or historical births for years corresponding to a specific condition (El Niño, position of the Bermuda high, etc.):

$$p_b \sim U[G(p_b)], \tag{2}$$

where $G(p_b)$ is a set of all candidate birth locations (first locations of each post-1944 track in the HURDAT dataset). In the applications presented in this paper we choose the birth location unconditionally from the candidate locations.

3) Sample the simulated track lifetime:

$$L \sim U(L_1, L_2), \tag{3}$$

where L is the entire duration of the cyclone in 6-h time steps, and L_1 and L_2 are the minimum and maximum total lifetimes of the tracks that lie in $B(p_b)$, where $B(p_b)$ is an area with a 2.5° radius around p_b, Tracks recorded in the years 1944–2011 are considered.

4) Define new position on the chosen track i (selected in step 2):

$$p_j = p(\tau_j), \tag{4}$$

where p_j is the position on the simulated track at iteration j and τ_j is the number of 6-h time steps represented by $\tau[C(\mathbf{x}^*)]$, the random amount of time in this state, to take along chosen track i moving forward from p_b. $\tau[C(\mathbf{x}^*)] \sim U(t^*, L)$, where t^* is the time elapsed on the current track and L is the simulated track length in 6-h steps.

5) The multivariate distance criteria $\boldsymbol{\theta}(\mathbf{x}^*)$ of the distance from the track to the current position, the vector difference in direction of movement relative to the current track, and the age (step number) relative to the current track were used to statistically capture and display the dynamical behavior differences of tropical cyclones born in different parts of the basin through the conditional probability density function, $f\{C(\mathbf{x}^*), \tau[C(\mathbf{x}^*)] \mid \boldsymbol{\theta}(\mathbf{x}^*)\}$. These variables are in essence surrogates for the physics of storm movement ensuring that jumps are made to similar neighbor tracks. Six clusters of North Atlantic tropical cyclone tracks were identified that display differing genesis locations, track shapes, intensities, life spans, landfalls, seasonal patterns, and trends (Nakamura et al. 2009). The relation of genesis location to life span (age) and preferred grouped paths (distance from the current position and the vector difference in the direction of movement) was considered on this basis.

Choose a track during the years 1851–2011 by drawing from "neighbor" tracks using the conditional density function defined through a product kernel density function as

$$f\{C(\mathbf{x}^*), \tau[C(\mathbf{x}^*)] \mid \boldsymbol{\theta}(\mathbf{x}^*)\} \propto k(u_{1ij})k(u_{2ij})k(u_{3ij}), \tag{5a}$$

where

$$K(u) = (1 - u^2)^2 \tag{5b}$$

is the bisquare kernel function and u_1, u_2, and u_3 represent distance measures in terms of different conditioning variables, as described below.

(i) The distance of a candidate historical track to the current track is

$$u_1 \propto \frac{D}{2.5(\pi/180)}; \quad D \le 2.5°, \tag{5c}$$

where D is the distance in degrees between neighbor track points in spherical geometry.

(ii) The orientation of a candidate historical track relative to the current track is

$$u_2 \propto \frac{V}{\mathrm{Max}(V)}; \quad V = (V_{xn} - V_{xi})^2 + (V_{yn} - V_{yi})^2, \tag{5d}$$

where $\mathrm{Max}(V)$ is the maximum wind vector difference over all tracks in knots, V_x is the instantaneous storm maximum wind speed in knots times the cosine of the angle between the current point and the next point on the track, V_y is the wind speed in knots times the sine of the angle, subscript i is the current track, and subscript n is the neighbor track.

(iii) The age of the candidate historical track relative to the current track is

$$u_3 \propto \frac{T}{\mathrm{Max}(T)}; \quad T = T_n - T_i, \tag{5e}$$

where T is age of the neighbor track, in 6-h steps, minus the age of the current track; $\mathrm{Max}(T)$ is the maximum T in 6-h steps over all tracks; subscript i refers to the current track; and subscript n refers to the neighbor track.

6) The 6-h steps remaining to the simulated storm end (R_j) are calculated as

$$R_j = L - a_j, \tag{6}$$

where L is the duration selected in step 3 and a_j is the age at iteration j.

7) The 6-h steps to take along chosen track i (S_j) are found by

$$S_j \sim U(1, R_j), \tag{7}$$

where R_j is selected in step 6.

8) The positions in 6-h steps on simulated track at iteration j are

$$p_j = p_b + S_j. \tag{8}$$

9) If $S_j = R_j$ then stop, else repeat steps 5–8.
10) Repeat steps 2–9, N (selected in step 1) times to complete a simulated season.

A numerical procedure for the efficient sampling and simulation of tracks was developed. A functional table was created that recorded the storm number, position on the track, latitude, longitude, number of time steps to the end of the track, the comparative vector of the storm direction, and wind speed. A second lookup table was created that listed the "neighbors" for all tracks for each position in each track. Neighbors are all points on other tracks within a 2.5° radius of the current location with a given probability based on the bisquare kernel function [(5b)] to move to that location based on distance, comparative vector, and age in 6-h steps as defined in step 5 in (5a)–(5e).

As an example, Fig. 3 shows the actual 2011 North Atlantic hurricane season tracks (top), and a HITS algorithm simulation of an arbitrary hurricane season plotted at the bottom. Figure 3 (bottom panel) shows that the tracks have more variation than the historical tracks, but they are still more coherent than the tracks from the Markov chain model in Fig. 1. Figure 3 (bottom panel) also illustrates the jumps of the simulated tracks as they move to neighbors; however, the accuracy of the model is assessed on the binned statistics in section 4 rather than the track paths. Movement to other tracks is realistic in terms of cyclone movement as track segments are selected by criteria of distance [(5c)], direction and speed vector [(5d)], and similar age [(5e)]. Consider a competing Markov chain model set up on a 1° × 1° or 2.5° × 2.5° discretization. Clearly, in that sort of a model one would have a jump of that magnitude in every time step and have highly discontinuous trajectories relative to observed trajectories or to those simulated by HITS.

4. Results

We present results for two types of tests. First, we present results for the statistics associated with the simulation of 1000 hurricane seasons. Next, we explore the conditional simulation of ensembles of tracks from different starting positions for real hurricanes (Sandy and Isaac) from 2012 that were not included in the model-fitting process.

The performance of the simulations for the 1000 seasons is judged through a variety of performance measures:

(i) comparing the average spatial distribution of the historical and simulated data,
(ii) comparing percentiles of the residence time in each grid box,
(iii) landfall statistics, and
(iv) comparing the frequency of 6-h periods of hurricane strength wind (>64 kt).

a. *Comparing the average spatial distribution of the historical and simulated data*

The track points for the post-1944 data at 6-h time steps are binned into boxes, and the count in each box is recorded. This number was divided by the total number of years of data (68) to compute mean annual values. The observations are heavily clustered along the curve of the parabolic sweep with a maximum off the coast of the southern United States (Fig. 4, top panel). Storm starting locations, being of particular interest, were also binned and plotted (Fig. 4, bottom panel). Births are clustered primarily in four regions covering approximately eight boxes, each having as many as 25 births over the 68 seasons.

Figure 5 shows the simulated corresponding figure of the mean 6-h periods (top panel) and births (bottom panel). The 6-h periods are slightly overestimated in the mean below 24°N and underestimated above that latitude. Exiting the tropics and entering the extratropics, cyclones are subjected to strong wind speeds and the 6-h observations are farther apart. In an alternate run (not shown), a 5° radius is employed rather than the 2.5° threshold. This change decreases the underestimation above 24°N; however, it greatly increases the overestimation below that latitude. Mean simulated starts (Fig. 5, bottom panel) slightly overestimate the tropical cyclone births off the coast of Africa, which may be the cause of the slight increase in the number of 6-h periods found there. Since these are randomly sampled unconditionally from the historical set, the difference is purely due to sampling variations.

b. *Comparing percentiles of the residence time in each grid box*

The spatial structure of the simulated tracks was assessed through a comparison of the number of 6-h time steps in each box relative to the historical data. Since we have 1000 simulations from HITS, we compute

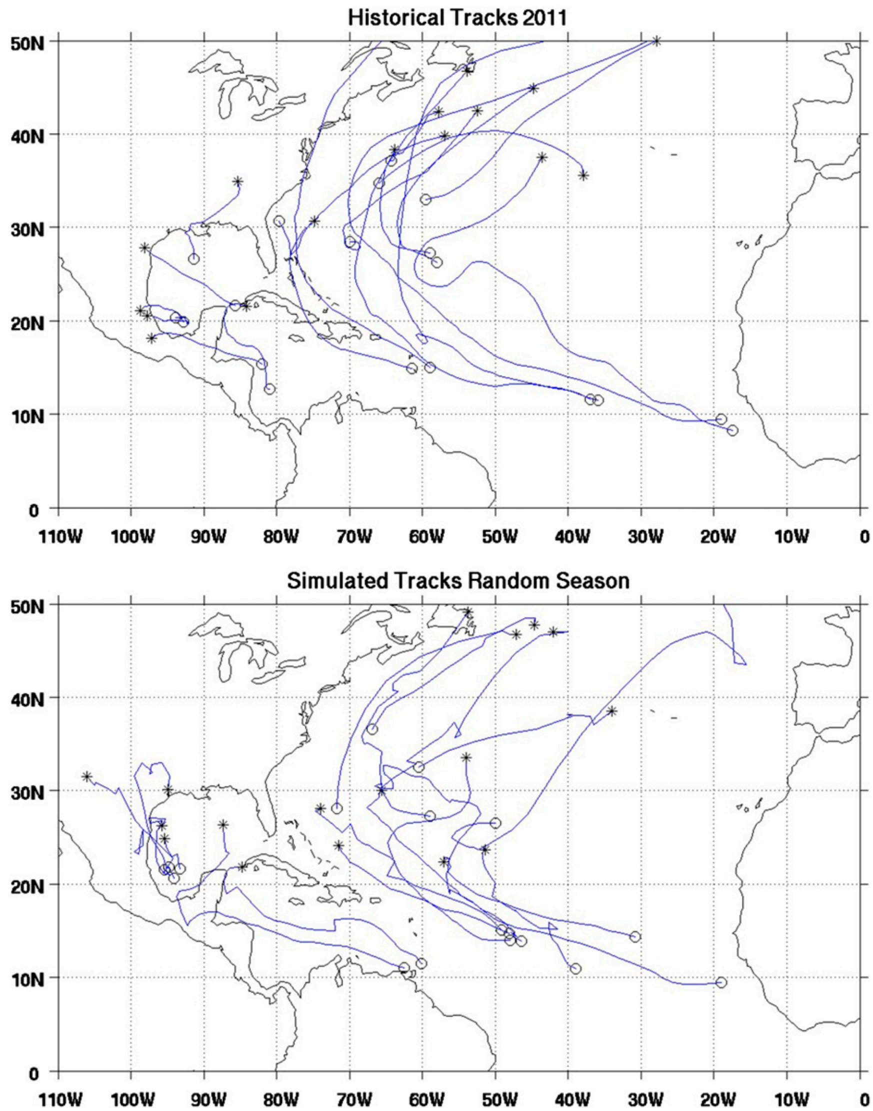

FIG. 3. (top) The 2011 North Atlantic hurricane season tracks and (bottom) a simulated North Atlantic hurricane season using HITS.

the percentile of the historical residence time relative to these simulations as the basis for comparison. In Fig. 6 the color blue indicates that the observations fall in the upper one-fifth percentile and green the upper quartile of the mean. Figure 6 shows that the simulation tends to underestimate the number of 6-h periods in data-sparse areas.

A comparison of the number of 6-h periods is offered in Fig. 7 for locations where mainland landfall is possible. A smoothed split violin plot (Fig. 7) shows the historical (blue, left) and simulated (red, right) kernel density plots for the number of 6-h time periods. In addition, the 25th, 50th, and 75th percentiles are marked with a line, and the mean values for historical (left) and simulated (right) results are printed along the x axis in Fig. 7. Labeling of the x-axis location is not inclusive, but provides us with a geographical marker in the box considered. The overall positive skew of the distribution of 6-h time periods is captured by the simulation with smoothing as expected from the larger sample size of the simulations. Also the extreme tail of the data (red thin lines extending upward from main portion of the data) is typically better populated as expected from the simulations. Boxes in which 6-h time periods are more prevalent (Florida, South Carolina) show pockets of the blue observed values on the red, long, thin simulated extreme tails. HITS allows the computation of events with potential return periods of 1000 years from the 1000 simulated years. This is in evidence from the extension of the tail relative to the observations.

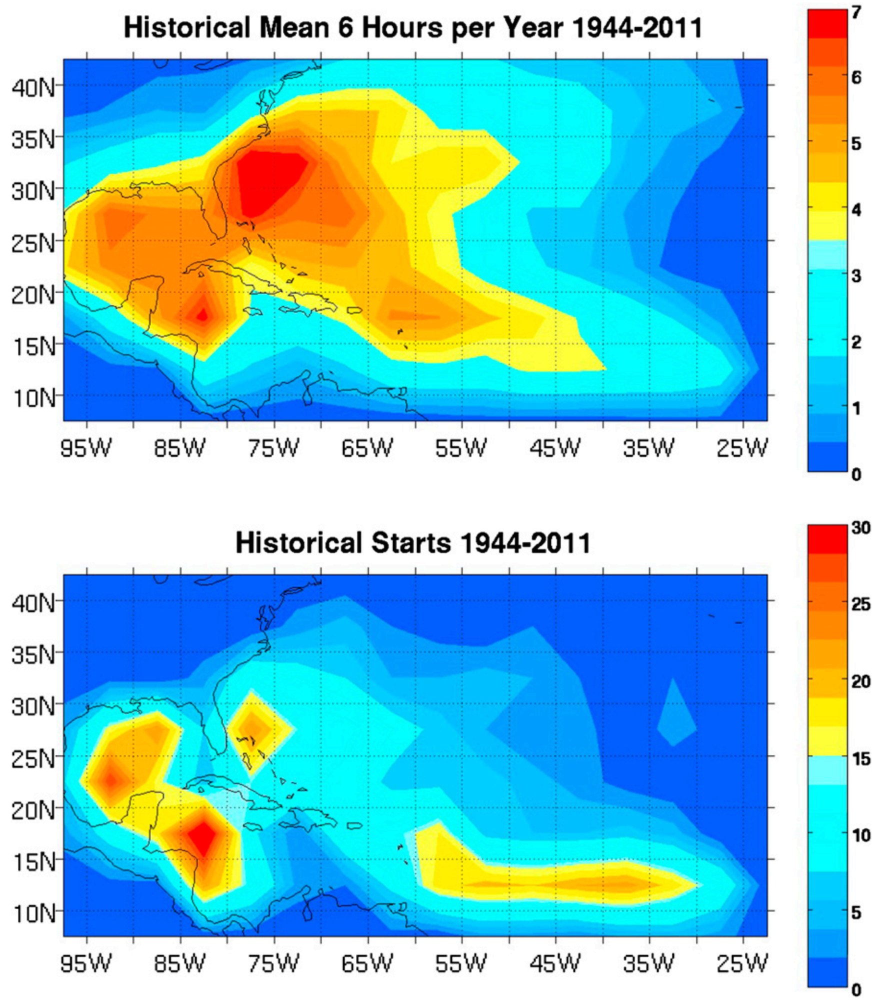

FIG. 4. (top) Average number of historical (1944–2011) hurricane 6-h time periods per year binned into 5° × 5° grid boxes and (bottom) number of historical tropical cyclone starts per box.

The historical distribution is occasionally multimodal while the simulated case is unimodal, reflecting the smoothing from the larger sample size. Differences in the underlying probability distributions of the historical and simulated results are tested using a two-sample Kolmogorov–Smirnov (KS) and Cramer–von Mises (CM) tests. Both the KS and CM tests are nonparametric and compare the location and shape of the empirical cumulative distribution functions of the two samples. All but one of the boxes (Virginia) pass the KS and CM tests at the 5% significance level indicating the same underlying probability distributions. Mean values of historical and simulated distributions along the *y* axis are similar, along with their distribution shapes. However, the median historical values fall between the 25th and 75th percentiles of the HITS simulations in all cases except for the low-populated Texas box.

c. *Landfall statistics*

A land–sea mask was created to indicate which boxes in the domain are considered mainland and which are ocean. From that it was possible to count the storms (both historical and simulated) as they crossed from ocean to the mainland, and vice versa. A smoothed split violin plot of historical (blue, left) and simulated (red, right) distributions of mainland landfalls is shown in Fig. 8. The width of the histogram is normalized to a maximum width equal to 0.9; the 25th, 50th, and 75th percentiles are marked with a line; and the mean values for historical (left) and simulated (right) results are printed along the *x* axis of Fig. 8. Distributions of landfalls are remarkably similar between the historical and simulated datasets. In a majority of the boxes the simulated extreme tail extends beyond the historical (Belize, Yucatan, Mexico, Louisiana, Florida, South Carolina,

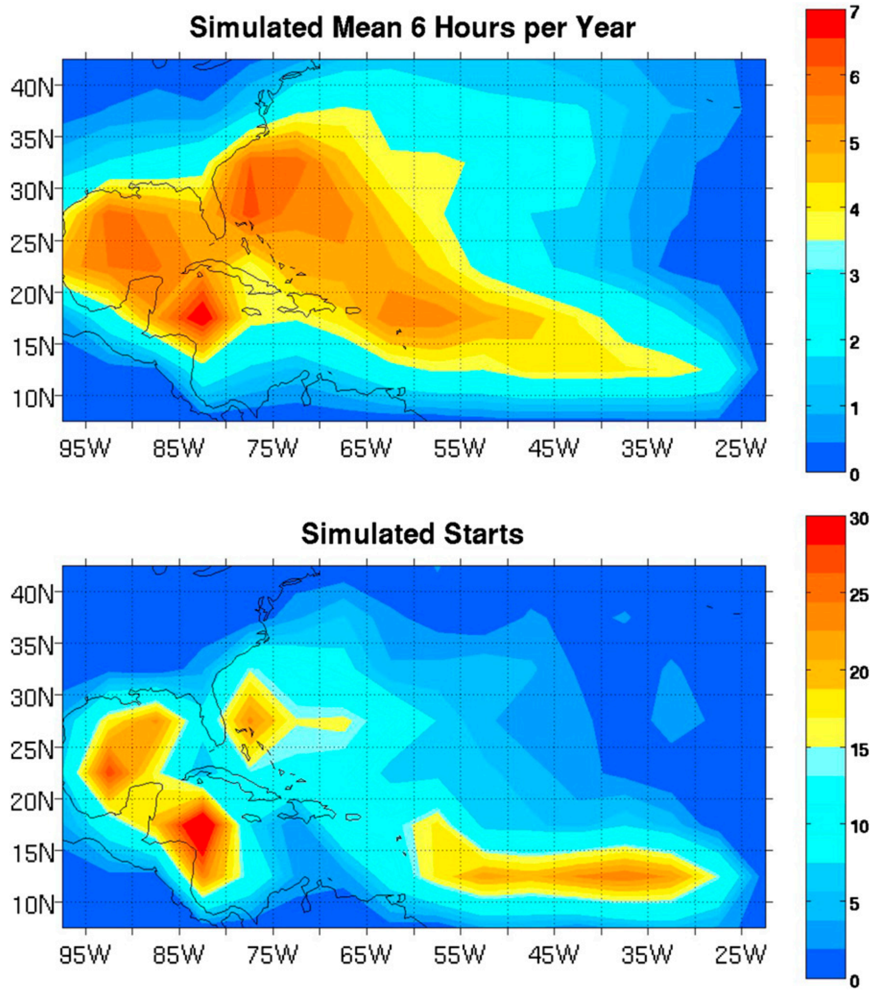

FIG. 5. (top) Average number of simulated hurricane 6-h time periods per year binned into $5° \times 5°$ grid boxes and (bottom) number of historical tropical cyclone starts per box times 68 to match the historical record of 1944–2011.

and Connecticut). The remaining landfall boxes are matched by the historical extreme value, except in Texas, where the result is underestimated. The Texas box appears in Fig. 6 as an area in which the low density of historical tracks leads to an underestimation of simulated tracks in those regions (median of the historical is above the 75th percentile of the simulation). As in the 6-h-per-year violin (Fig. 7), all but one of the boxes (Virginia) passes the KS and CM tests at the 5% significance level, indicating that the underlying probability distributions of the historical and simulated datasets are the same.

d. *Comparing frequency of 6-h periods of hurricane strength wind (>64 kt)*

Wind speed information (as well as any other track information in the HURDAT dataset) associated with the historical tracks is retained in the simulated tracks. For 6-h time periods of hurricane strength (33 m s^{-1}, 64 kt, or 74 mi h^{-1}) and above, a smoothed split violin plot (Fig. 9) shows the historical (left, blue) and simulated (right, red) distributions in mainland landfall areas. The hurricane strength statistic is a subset of the 6-h time periods per year (Fig. 7), but the distribution of counts is different for the two: smoother and shorter simulation tails. The historical and simulated means are closer in value for the hurricane-strength cases (Fig. 9) than for the results for the 6-h time periods per year (Fig. 7), although the overall numbers of occurrences are reduced. All boxes pass the KS and CM tests at the 5% significance level indicating that the underlying probability distributions of the historical and simulated results are the same.

e. *2012 hurricanes*

Since HITS was fit using the HURDAT dataset of 1851–2011, an ‘‘out of’’ sample of HITS on 2012 tracks was made by selecting different positions on a hurricane

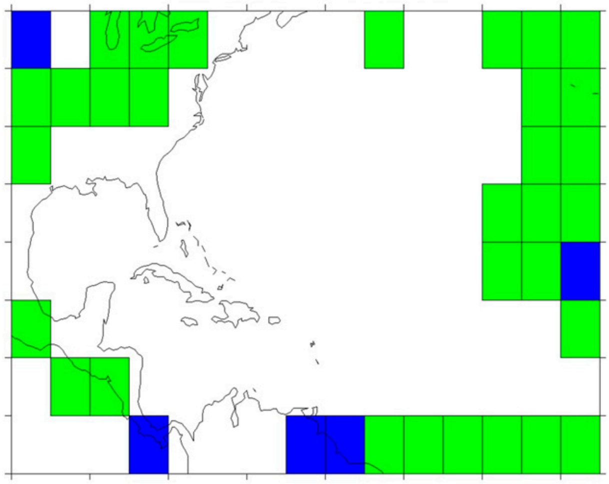

FIG. 6. The spatial structure of the 1000 simulated tracks shown by the percentiles of the count of 6-h time steps in each box compared to the historical record (1944–2011). The color blue implies that the observations fall in the upper one-fifth percentile, and green indicates the upper quartile.

track from which to simulate. This is a different way of applying the same model. Instead of looking at long-term simulations, one looks at a given hurricane track at a particular stage and, using the conditional distribution of trajectory and intensity given a position on the track, generates forward simulations. This was applied to several historical hurricanes with similar results. Here, we present the comparisons for two recent hurricanes: Isaac and Sandy from 2012. In each case, different starting positions for the conditional simulation were considered prior to landfall, and 1000 simulations were performed for each starting position. These are compared with the NOAA hurricane forecasts from the same locations.

Text files of latitude, longitude, and wind speed measurements every 6 h, as in the HURDAT dataset, were taken from the Atlantic Hurricane Track Map and Images Internet page of The Johns Hopkins University (http://fermi.jhuapl.edu/hurr/index.html). The 5-day track forecast, uncertainty cone, and watch/warning images were obtained from National Weather Service's National Hurricane Center, while the near-surface daily wind speed over land and ocean were taken from the NCEP–NCAR reanalysis dataset's (Kistler et al. 2001) 0.9950 sigma level.

1) SANDY

Sandy was a devastating storm in 2012, making landfall in Jamaica as a category 1 storm on the Saffir–Simpson hurricane wind scale, in Cuba as a category 3, and in southern New Jersey as a post–tropical cyclone, with a significant storm surge in the mid-Atlantic and New England states (Blake et al. 2013). Figures 10a–d show, respectively, the HITS hurricane wind strength area in 6-h time periods per year, the maximum sustained winds from 26 to 31 October 2012, the simulated mainland landfalls, and the watch/warning images from the National Hurricane Center (d) for at 1700 eastern daylight time (EDT) on 26 October 2012. Black circles in Figs. 10a–c mark the actual path of Hurricane Sandy with the large black X in each of the panels indicating where the simulation was started.

On 26 October, 3 days before landfall in southern New Jersey, Sandy was off the Florida coast with HITS showing high values of hurricane strength 6-h time periods and mainland landfalls along the eastern coast. The actual path of Sandy in Fig. 10a is within the HITS hurricane strength wind area over the ocean. The solid contour in Fig. 10b indicates Sandy's maximum sustained hurricane strength winds and is similar to the size and shape of the simulated hurricane wind area in Fig. 10a. HITS mainland landfalls in Fig. 10c have bull's-eyes on Florida and the mid-Atlantic. On 27 October (not shown), HITS no longer has the maximum landfall occurring in Florida, as the hurricane is farther north and east by then. The NHC forecast in Fig. 10d shows landfall in Delaware, south of the eventual landfall location.

2) ISAAC

Hurricane Isaac passed over the Lesser Antilles, Haiti, and eastern Cuba as a tropical storm, and intensified to a category 1 hurricane before making landfall in southeastern Louisiana, causing storm surge and inland flooding across southeastern Louisiana and southern Mississippi (Berg 2013). Figures 11a–d show, respectively, the HITS hurricane wind strength area in 6-h time periods per year, the maximum sustained winds from 25 to 30 Aug 2012, the simulated mainland landfalls, and the watch/warning imagery from the National Hurricane Center at 0500 EDT 25 August 2012. Black circles in Figs. 11a–c mark the actual path of Hurricane Isaac with the large black X in each of the panels indicating where the simulation was started.

On 25 August, Isaac was over Cuba and the actual path in Fig. 11a was within the HITS hurricane strength wind area over the ocean. The solid contour in Fig. 11b indicates Isaacs's maximum sustained hurricane strength winds and is similar to the size and shape of the simulated hurricane wind area in Fig. 11a. HITS mainland landfalls in Fig. 11c have a large value over Florida with smaller ones along the Gulf Coast, the East Coast, and even over Central America. On 27 August (not shown), HITS has a greater probability of landfall on the

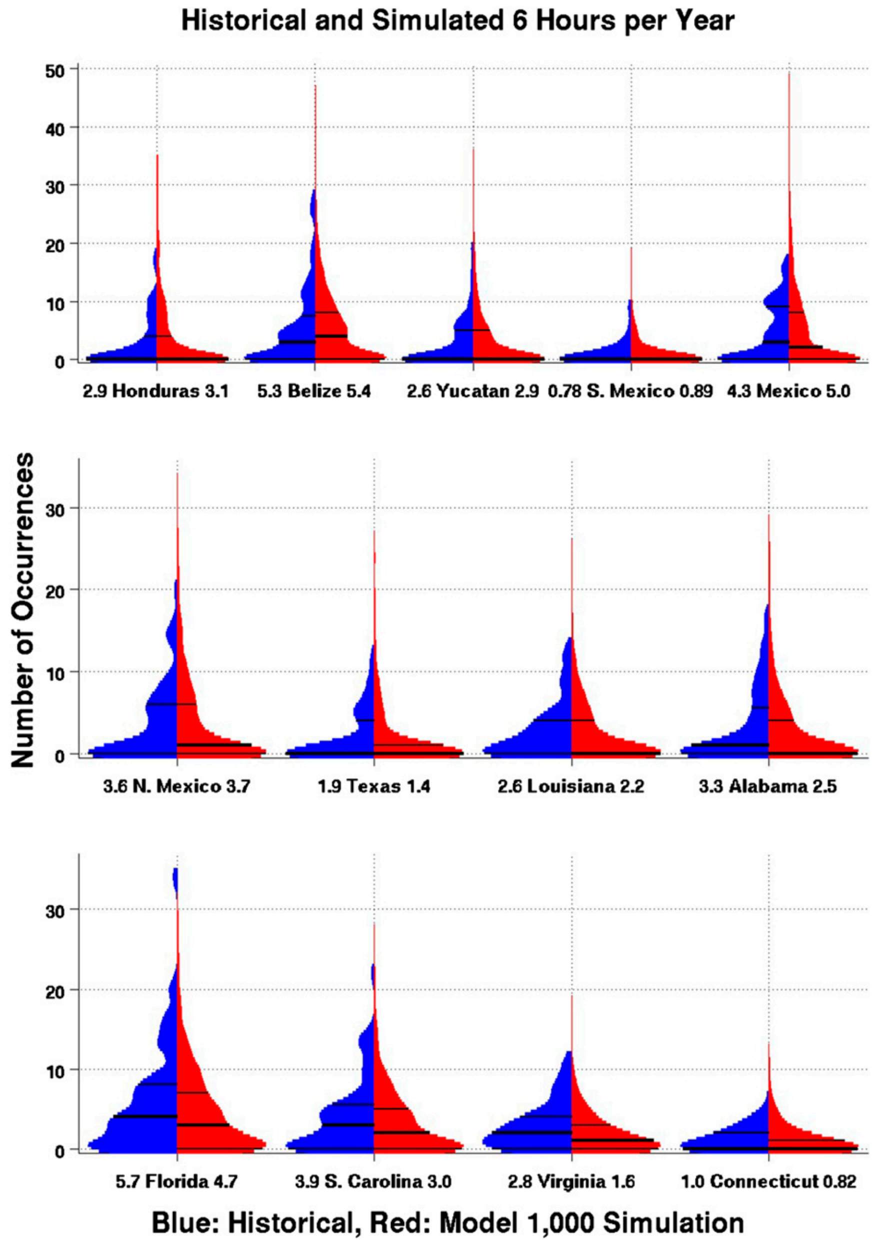

FIG. 7. Kernel density estimation split violin plot of observed (blue) and simulated (red) 6-h time periods per year for landfall areas. Mean values for observed and simulated results are located along the x axis.

Gulf Coast as it moves into the Gulf of Mexico. The NHC forecast in Fig. 11d shows landfall in western Florida, east of the actual Louisiana landfall.

5. Summary and discussion

A new, nonparametric tropical cyclone track simulator that is motivated by the observation that Markovian models tend to be diffusive relative to historical observations was presented. The basic idea, inspired by a nonhomogeneous hidden Markov renewal model formulation, considers conditional distributions of latent states that generate hurricane tracks, through resampling along a track using a kernel density function with k-nearest neighbor bandwidth, applied to selected track attributes. Samples from this nonparametric conditional distribution function lead to the transition to a new latent state that is realized as a shift to historical hurricane track at each transition location. For model development, space is considered to be continuous, with no gridding, while

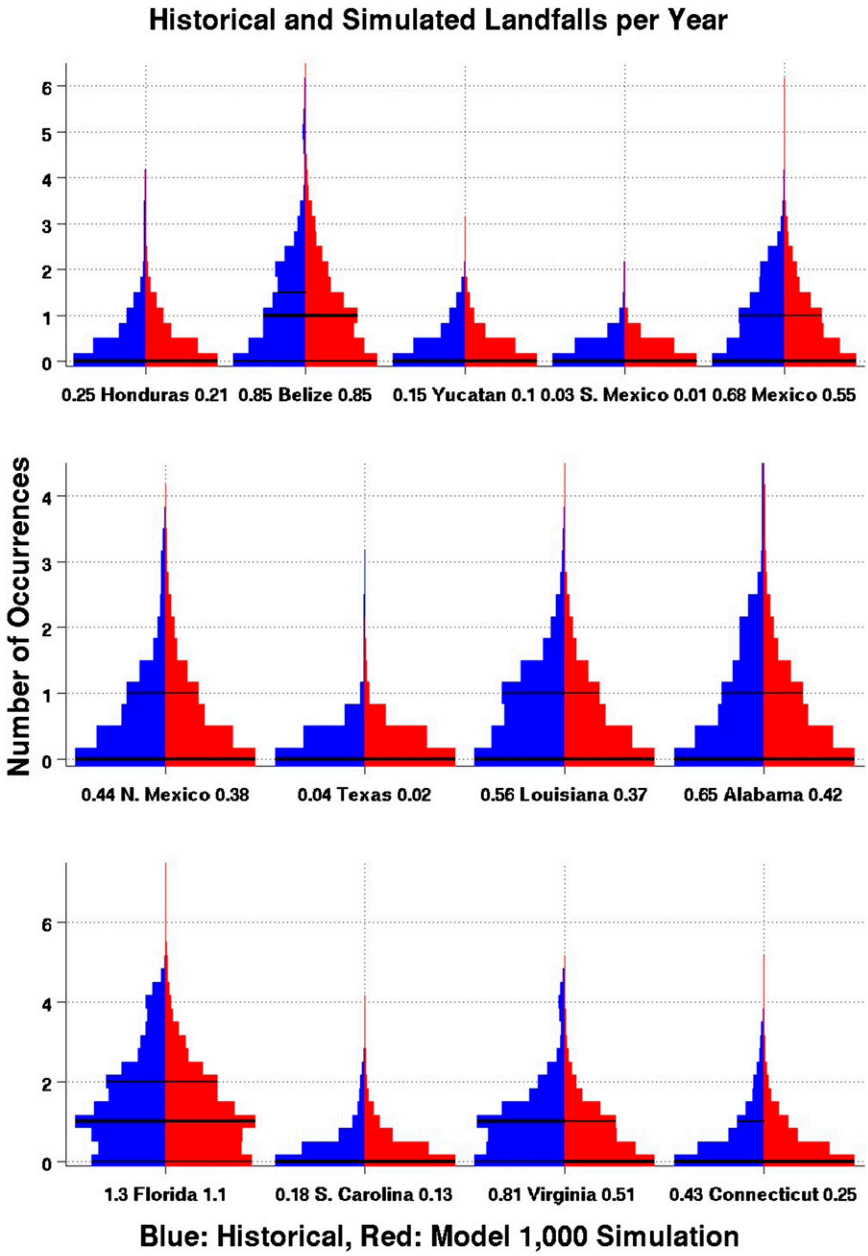

FIG. 8. As in Fig. 7, but for the number of tropical cyclone landfalls per year.

time is discrete following 6-h steps. The local tracks at a point are considered candidate states, and a transition to one of them is selected based on the distance between tracks, track orientation, and the age of each of the tracks. The last variable allows a consideration of the past history of each of the tracks and hence discriminates between tracks that may have been just born near that location versus ones that were born quite a bit farther away. Thus, the length of time in a past state is accounted for and the length of time to spend in the new state is simulated, as in a Markov renewal model. The residence time, spatial distribution, and landfall densities of the simulations are well simulated and historical track information is carried through, allowing for different aspects of risk assessment.

Performance of the HITS simulated tracks was evaluated over $5° \times 5°$ boxes for a number of statistics. While some biases are evident in the areas poorly sampled in the historical database, it is clear that the simulations preserve the essential attributes of residence time, spatial patterns, and landfall, especially for the stronger wind thresholds: the 6-h periods of hurricane strength

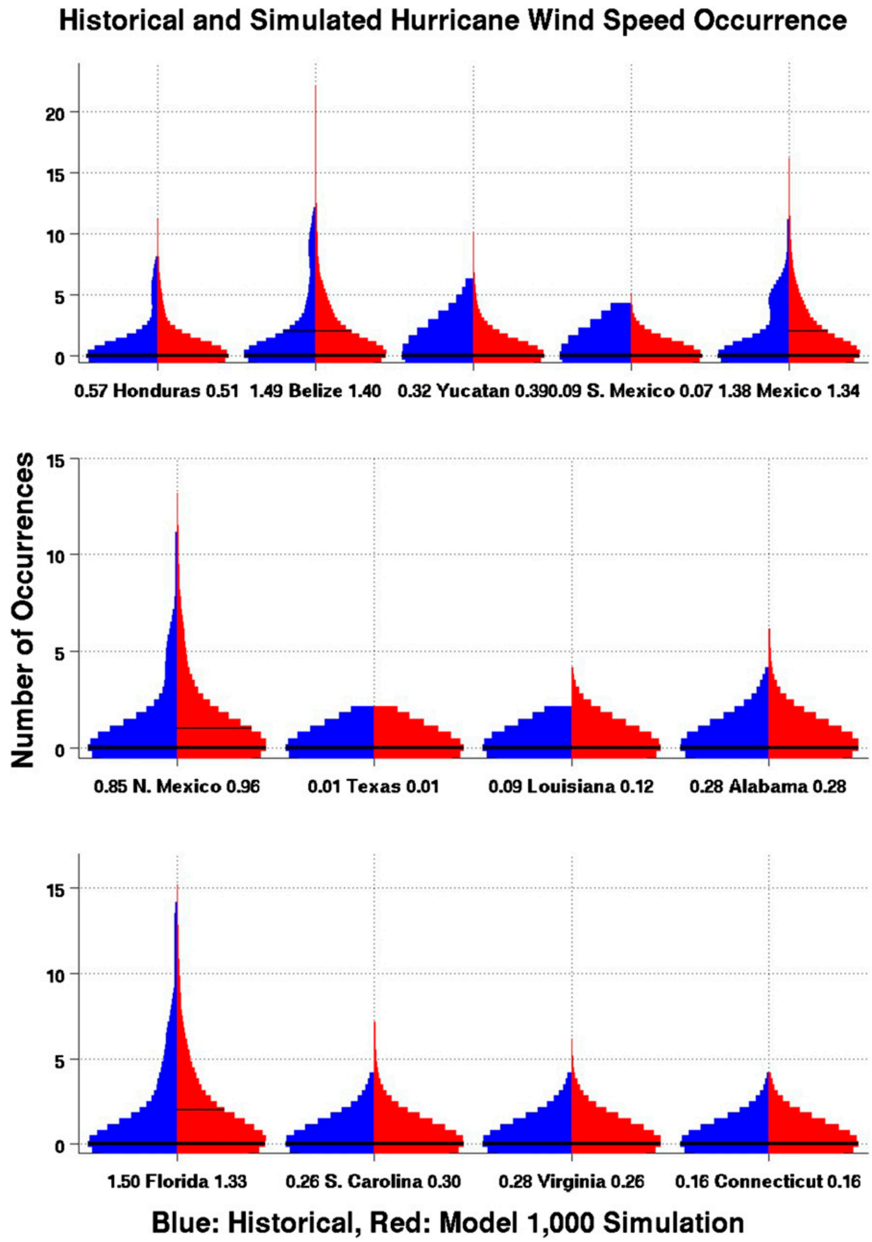

FIG. 9. As in Fig. 7, but for hurricane strength winds.

winds for all mainland landfall boxes passed the KS and CM tests at the 5% significance level, indicating that the historical and simulated results were from the same distribution. Even though the model is based directly on the historical record and is nonparametric, an extension of the tail probability distribution and smoothing of the probability distribution of the statistics of interest is seen relative to the historical data. Similarly, conditional simulations of historical tracks showed propagation dynamics that, relative to the point from which they are started, have performance similar to those produced by dynamical models that are in use for near-real-time tropical cyclone forecasts. We do not suggest HITS as a forecast model since none of the essential physics is modeled at all. However, it seems that the information contained in the historical tracks does contain enough of the location-relevant physics such that the model that simulates tracks based on geometrical similarity criteria is able to do a conditional simulation of the tracks from different locations. The ability of HITS to simulate individual tracks that are based on historical tracks is a novel feature of this model.

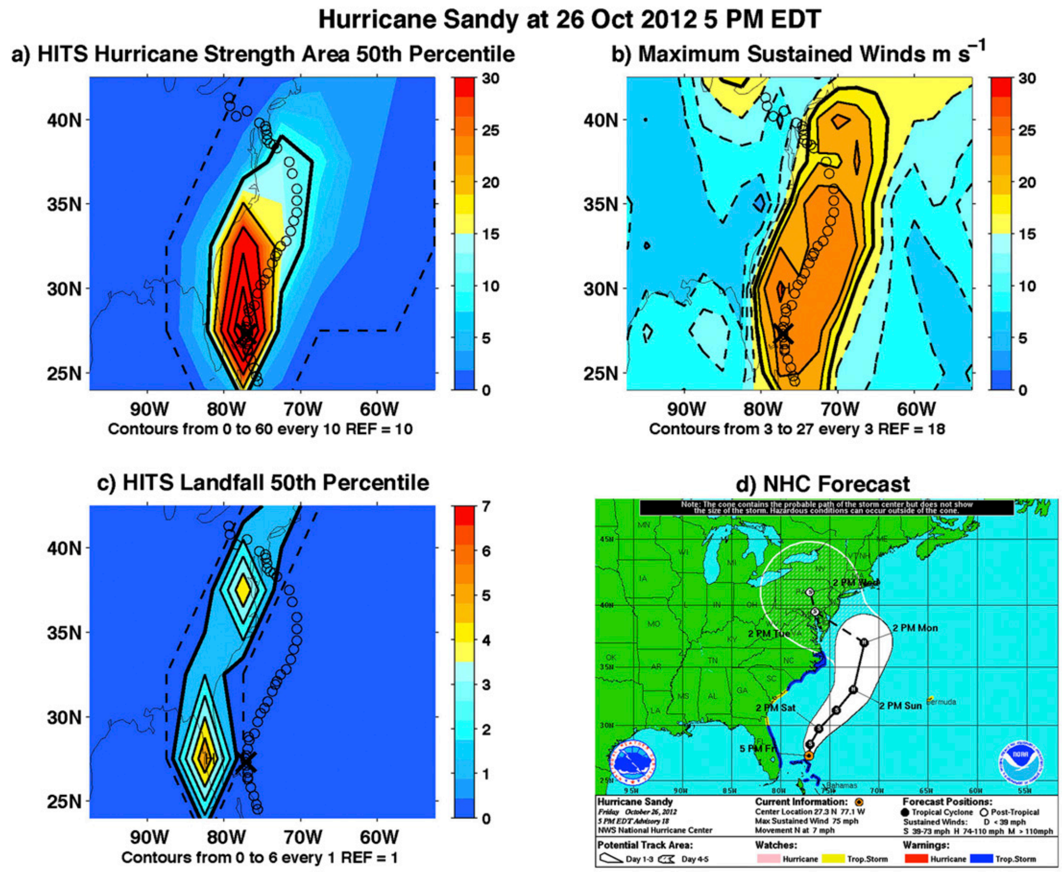

FIG. 10. For Hurricane Sandy, (a) simulated hurricane wind strength area in 6-h time periods per year, (b) maximum sustained winds from 26 Oct to 31 Oct 2012, (c) simulated mainland landfalls, and (d) watch/warning image from the National Hurricane Center at 1700 EDT 26 Oct 2012. Black circles in (a)–(c) indicate the actual path of Hurricane Sandy and a large black X indicates where the simulation was started.

Several track simulation modelers have divided up the Gulf and U.S. coasts and compared their model landfall results with the historical HURDAT dataset [Figs. 4 and 5 and Table 1 in Hallegatte (2007), Fig. 18 in Hall and Jewson (2007), Fig. 3 in Vickery et al. (2000), and Table 1 in Rumpf et al. (2009)], The areas are different for all models, so they cannot be compared directly. They all use different statistical tools to judge the "goodness" of the fit.

Hall and Jewson (2007) also use the number of 6-h tropical cyclone positions per area in their Fig. 15. Emulating the "Z score" (normalized probability of historical minus simulated mean divided by historical) in their Fig. 15d, all of the HITS boxes were between -1 and $+1$ except for those where historical and simulated counts were zero, leaving an undefined value. The mean normalized probability was 0.063. Although it is important to compare the mean (or median) of simulated and historical data, this analysis emphasized comparing the shape of the entire distribution as extreme events appear on the tail end. Research has shown that not only do tropical cyclone distributions display a heavy tail (Figs. 7–9), but hurricane damage is also heavily tailed (Katz 2002).

In sensitivity testing of the HITS model, the following approaches were examined:

- A 5° radius of neighbor points was tried rather than the 2.5° (given as D). This larger radius sampled among unlike populations of tracks in the tropics and was abandoned.
- Several ways of computing the direction vector were attempted: with previous, current, or future points, as vector differences, or as distance angle vectors. Computing both angles and distances with future points (distance or angle needed to jump to next track segment) gave the best results.
- The median of track lifetimes was used rather than random draw (given as L). Simulated track length was unrealistic using the median, as shorter and longer tracks were not represented.

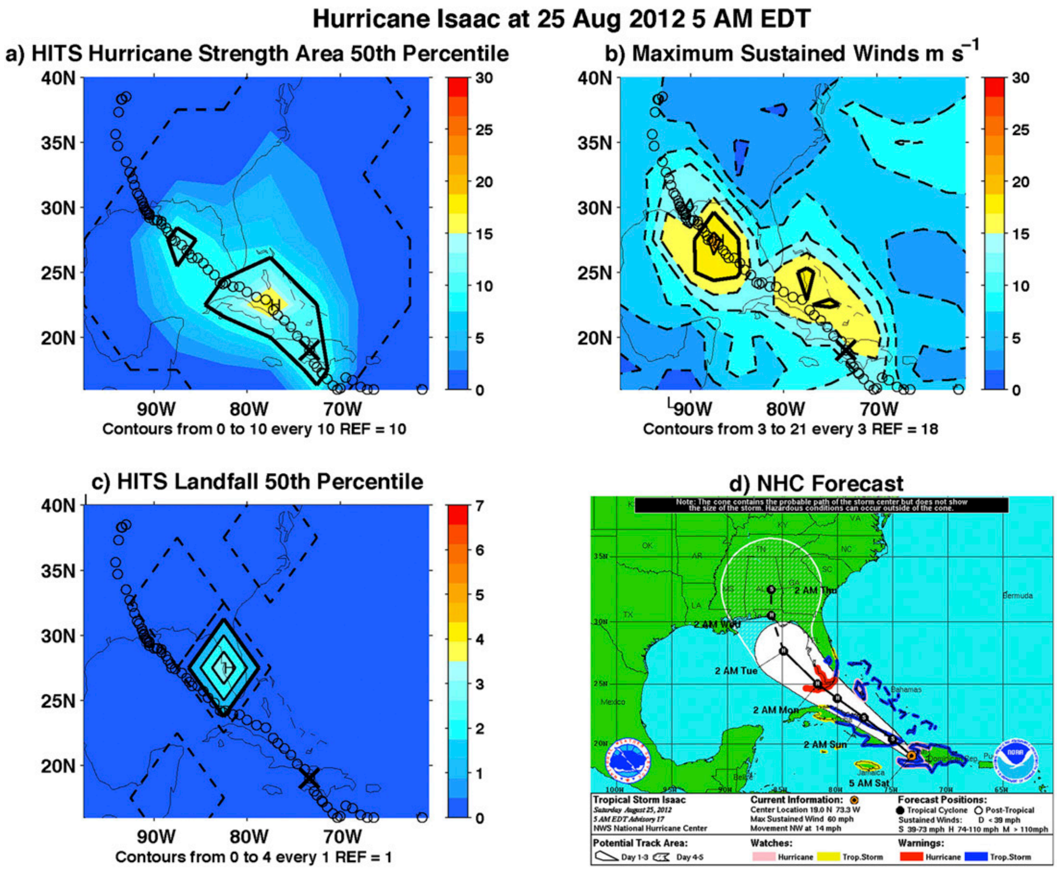

FIG. 11. For Hurricane Isaac, (a) simulated hurricane wind strength area in 6-h time periods per year, (b) maximum sustained winds from 25 to 30 Aug 2012, (c) simulated mainland landfalls, and (d) watch/warning image from the National Hurricane Center at 0500 EDT 25 Aug 2012. Black circles in (a)–(c) indicate the actual path of Hurricane Isaac and a large black X indicates where the simulation was started.

- Use of only post-1944 data was explored. The best results came from using all available data. If the quality of the data is a concern, selection of those tracks can be down weighted. Behavior of a system with determinism like the paths of North Atlantic hurricanes is best studied with all available information on past behavior.

Our future work plan includes running the model backward to determine where all landfalling storms in a particular box started. We also plan to explicitly consider conditioning on large-scale climate variables to see if interannual variability in hurricane counts and tracks can be properly simulated. As clustering results of North Atlantic hurricane tracks have shown groupings that display differing genesis locations, track shapes, intensities, life spans, landfalls, seasonal patterns, and trends (Nakamura et al. 2009), selecting the birth location based on climate state would also impact the resulting tracks and probabilities.

Acknowledgments. Our work was supported by NSF Grant AGS-1003417. Research by YK and UL on this project was partially funded by the National Oceanic and Atmospheric Administration's RISA Program, Award NA10OAR4310212, for the "Consortium for Climate Risk in the Urban Northeast (CCRUN)." Analysis was made possible by Suzana Camargo's Matlab-converted HURDAT dataset.

APPENDIX

Alphabetical List of Terms in the HITS Conceptual Model

b Subscript indicating first recorded location of track

B Circular area of 2.5° around the first recorded location of the selected track

C Track or curve

D	Distance between neighbor tracks in spherical geometry
$f(\,)$	Indicates function of ( )
G	Set of all recorded track locations within B
i	Subscript of position on the historical track
j	Subscript of position of the simulated track
k, l	Indices of latent states
$K(\,)$	Kernel function
L	Simulated track lifetime
n	Subscript of position of neighbor track
N	Number of tracks in the simulated season
p	Position on the simulated track as number of 6-h steps
R	Number of 6-h steps remaining until simulated storm end
S	Number of 6-h steps to take along the historical track
$\tau(\mathbf{x}^*)$	Time spent in a latent state at a transition from $\mathbf{x}^*$
$\boldsymbol{\theta}(\mathbf{x}^*)$	Vector of covariates for state transition from $\mathbf{x}^*$
T	Relative age of the neighbor track
V	Wind vector difference
$\mathbf{x}^*$	Position at state transition location
$\mathbf{x}$	Location vector on a track

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